In order to study smoothing methods it is of paramount importance to use made-up data where the true model is known. Certainly in the usual application no representation of the true underlying function or surface is known, but if a method approximates a variety of surface behavior faithfully, then we expect it to give reasonable results in other cases. Consequently, the implemented methods were tested for accuracy on artificial data sets constructed by adding Gaussian (normal) noise and/or outliers to different test functions.

Our test suite consists of 15 functions, some of them with multiple features and abrupt transitions. While all the test functions are continuous, some of them have a discontinuous derivative at one point, what make smoothing and the derivatives estimation quite challenging.

A dedicated GUI allows users to easily choose one of the functions, to add noise, outliers, and placement of the input points. Nodes placement can be varied from a regular grid to a complete spatial randomness.

As doFORC is mainly dedicated to FORC diagrams calculation, the test functions $f_3, f_4, f_5,$ and f_6 provide FORCs for different types of Preisach distributions. For all other test functions the data can be generated either in a rectangular domain or in a FORC style in the $h_{\text{applied}} \geq h_{\text{reversal}}$ half-plane.

Test Functions	
1	Probability density function (PDF) of the bivariate normal distribution: $P_{\text{BVN}}(x, y, \mu_x, \mu_y, \sigma_x, \sigma_y, \rho) =$ $= \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\rho^2}} \exp\left\{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{x-\mu_x}{\sigma_x}\right)^2 + \left(\frac{y-\mu_y}{\sigma_y}\right)^2 - 2\rho\left(\frac{x-\mu_x}{\sigma_x}\right)\left(\frac{y-\mu_y}{\sigma_y}\right)\right]\right\}$ $\rho = \text{cor}(x, y) =$correlation coefficient; $-1 < \rho < 1$ $\mu_x, \ \mu_y =$ means $\sigma_x, \ \sigma_y =$ standard deviations; $\sigma_x > 0$; $\sigma_y > 0$
2	Lower cumulative distribution function (CDF) of the bivariate normal distribution: $CDF_{\text{BVN}}(x, y, \mu_x, \mu_y, \sigma_x, \sigma_y, \rho) = \int_{-\infty}^{x} d\xi \int_{-\infty}^{y} d\eta \cdot P_{BVN}(\xi, \eta, \mu_x, \mu_y, \sigma_x, \sigma_y, \rho)$
3	$\text{Preisach}_{\text{BVN}}(x \geq y, y, \mu_1, \mu_2, \sigma_1, \sigma_2, \rho) = 1 - \int_{x}^{\infty} d\xi \int_{y}^{\xi} d\eta \cdot P_{\text{BVN}}\left(\frac{\xi-\eta}{2}, \frac{\xi+\eta}{2}, \mu_1, \mu_2, \sigma_1, \sigma_2, \rho\right)$ $\mu_1 =$ mean of the coercive field distribution $\mu_2 =$ mean of the interaction field distribution $\sigma_1 =$ standard deviation of the coercive field distribution $\sigma_2 =$ standard deviation of the interaction field distribution
4	$\text{Preisach}_{\text{BVGN}}(x \geq y, y, \mu_1, \mu_2, \sigma_1, \sigma_2, \rho, \beta) = 1 - \int_{x}^{\infty} d\xi \int_{y}^{\xi} d\eta \cdot P_{\text{BVGN}}\left(\frac{\xi-\eta}{2}, \frac{\xi+\eta}{2}, \mu_1, \mu_2, \sigma_1, \sigma_2, \rho, \beta\right)$

$$P_{\text{BVGN}}\left(u, v, \mu_1, \mu_2, \sigma_1, \sigma_2, \rho, \beta\right) =$$

$$= \frac{1}{2^{\frac{1}{\beta}} \pi \sigma_1 \sigma_2 \sqrt{1-\rho^2}\; \Gamma\left(\frac{1}{\beta}\right)}$$

$$\times \exp\left\{ -\frac{1}{2} \left(\frac{1}{1-\rho^2} \left[\left(\frac{u-\mu_1}{\sigma_1} \right)^2 + \left(\frac{v-\mu_2}{\sigma_2} \right)^2 - 2\rho \left(\frac{u-\mu_1}{\sigma_1} \right) \left(\frac{v-\mu_2}{\sigma_2} \right) \right] \right)^{\beta} \right\}$$

$= \text{PDF of the bivariate generalized normal distribution}$

$\Gamma\left(z\right) = \text{gamma function}$

$\beta = \text{shape parameter};\;\; \beta > 0$

Observation: P_{BVN} is obtained as a particular case for $\beta = 0$.

5

$$\text{Preisach}_{\text{BVSN}}\left(x \geq y, y, \mu_1, \mu_2, \sigma_1, \sigma_2, \rho, \delta_{11}, \delta_{12}, \delta_{21}, \delta_{22}\right) =$$

$$= 1 - \int_x^{\infty} d\xi \int_y^{\xi} d\eta \cdot P_{\text{BVSN}}\left(\frac{\xi-\eta}{2}, \frac{\xi+\eta}{2}, \mu_1, \mu_2, \sigma_1, \sigma_2, \rho, \delta_{11}, \delta_{12}, \delta_{21}, \delta_{22} \right)$$

$$P_{\text{BVSN}}\left(u, v, \mu_1, \mu_2, \sigma_1, \sigma_2, \rho, \delta_{11}, \delta_{12}, \delta_{21}, \delta_{22}\right) =$$

$$= \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}\left[\frac{1}{2} - \frac{1}{2\pi}\arccos(\tilde{\rho})\right]}$$

$$\times \exp\left\{ -\frac{1}{2\left(1-\rho^2\right)} \left[\left(\frac{u-\mu_1}{\sigma_1} \right)^2 + \left(\frac{v-\mu_2}{\sigma_2} \right)^2 - 2\rho \left(\frac{u-\mu_1}{\sigma_1} \right) \left(\frac{v-\mu_2}{\sigma_2} \right) \right] \right\}$$

$$\times \Phi\left[\delta_{11}\left(u-\mu_1\right) + \delta_{11}\left(v-\mu_2\right)\right]\; \Phi\left[\delta_{21}\left(u-\mu_1\right) + \delta_{21}\left(v-\mu_2\right)\right]$$

$$= P_{\text{BVN}}\left(u, v, \mu_1, \mu_2, \sigma_1, \sigma_2, \rho\right) \frac{\Phi\left[\delta_{11}\left(u-\mu_1\right) + \delta_{12}\left(v-\mu_2\right)\right]\; \Phi\left[\delta_{21}\left(u-\mu_1\right) + \delta_{22}\left(v-\mu_2\right)\right]}{\frac{1}{2} - \frac{1}{2\pi}\arccos(\tilde{\rho})}$$

$= \text{PDF of the bivariate skew normal distribution}$

$\delta_{11}, \delta_{12}, \delta_{21}, \delta_{22} = \text{skewness parameters}$

$$\Phi\left(\tau\right) = \int_{-\infty}^{\tau} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{t^2}{2}\right) dt = \frac{1}{2}\left[1 + \text{erf}\left(\frac{\tau}{\sqrt{2}}\right)\right] = \text{CDF of the univariate standard normal distribution}$$

$$\tilde{\rho} = \frac{\delta_{21}\delta_{11}\sigma_1^2 + \delta_{22}\delta_{12}\sigma_2^2 + \left(\delta_{12}\delta_{21} + \delta_{22}\delta_{11}\right)\sigma_1\sigma_2\rho}{\sqrt{\left(1 + \delta_{11}^2\sigma_1^2 + 2\delta_{11}\delta_{12}\sigma_1\sigma_2\rho + \delta_{12}^2\sigma_2^2\right)\left(1 + \delta_{21}^2\sigma_1^2 + 2\delta_{21}\delta_{22}\sigma_1\sigma_2\rho + \delta_{22}^2\sigma_2^2\right)}}$$

Observation: P_{BVN} is obtained as a particular case for $\delta_{11} = \delta_{12} = \delta_{21} = \delta_{22} = 0$.

6	Preisach $_{\text{BVSHN}}(x \geq y, y, \mu_1, \mu_2, \sigma_1, \sigma_2, \rho, \alpha_1, \alpha_2) =$ $= 1 - \int_x^\infty d\xi \int_y^\xi d\eta \cdot P_{\text{BVSHN}}\left(\frac{\xi-\eta}{2}, \frac{\xi+\eta}{2}, \mu_1, \mu_2, \sigma_1, \sigma_2, \rho, \alpha_1, \alpha_2\right)$ $P_{\text{BVSHN}}(u, v, \mu_1, \mu_2, \sigma_1, \sigma_2, \rho, \alpha_1, \alpha_2) =$ $= \frac{4}{\alpha_1\,\alpha_2} \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \exp\left\{-\frac{1}{2(1-\rho^2)}\left[\left(\frac{2}{\alpha_1}\sinh\left(\frac{u-\mu_1}{\sigma_1}\right)\right)^2 + \left(\frac{2}{\alpha_2}\sinh\left(\frac{v-\mu_2}{\sigma_2}\right)\right)^2 -2\rho\left(\frac{2}{\alpha_1}\sinh\left(\frac{u-\mu_1}{\sigma_1}\right)\right)\left(\frac{2}{\alpha_2}\sinh\left(\frac{v-\mu_2}{\sigma_2}\right)\right)\right]\right\}$ $\times \cosh\left(\frac{u-\mu_1}{\sigma_1}\right) \cosh\left(\frac{v-\mu_2}{\sigma_2}\right)$ = PDF of the bivariate sinh-normal distribution α_i = shape parameters; $\alpha_i > 0$ Observation: for $\alpha > 2$ the P_{BVSHN} is multimodal.
7	$f_7(x, y, x_c, y_c) = \tanh(x - x_c)\tanh(y - y_c)$
8	$f_8(x, y, x_c, y_c) = \sin(x - x_c)\sin(y - y_c)$
9	$f_9(x, y, x_c, y_c) = \operatorname{sinc}(x - x_c)\operatorname{sinc}(y - y_c) = \frac{\sin(x-x_c)}{x-x_c}\frac{\sin(y-y_c)}{y-y_c}$
10	$f_{10}(x, y, x_c, y_c) = \sin\left(\sqrt{(x-x_c)^2 + (y-y_c)^2}\right) \equiv \sin r$ Observation: f_{10} is smooth except for a first derivative discontinuity at the point (x_c, y_c).
11	$f_{11}(x, y, x_c, y_c) = \operatorname{sinc}\left(\sqrt{(x-x_c)^2 + (y-y_c)^2}\right) \equiv \operatorname{sinc} r$
12	$f_{12}(x, y, x_c, y_c) = \sqrt{(x-x_c)^2 + (y-y_c)^2} \equiv r$ Observation: f_{12} is smooth except for a first derivative discontinuity at the point (x_c, y_c).
13	$f_{13}(x, y, x_c, y_c) = \exp\left[-0.2\sqrt{(15(x-x_c))^2 + (20(y-y_c))^2}\right]\cos\left(\sqrt{(15(x-x_c))^2 + (20(y-y_c))^2}\right)$ Observation: f_{13} is smooth except for a first derivative discontinuity at the point (x_c, y_c).
14	$f_{14}(x, y, x_c, y_c) = \left(5^3\exp[-5u]\exp[-5v]\right)\left(\frac{1}{(1+\exp[-5u])(1+\exp[-5v])}\right)^5$ $\times\left(\exp[-5u] - \frac{2}{1+\exp[-5u]}\right)\left(\exp[-5v] - \frac{2}{1+\exp[-5v]}\right)$ where $u = x - x_c$; $v = y - y_c$
15	$f_{15}(x, y, x_c, y_c) = 3(u-1)^2\exp\left[-u^2 - (v+1)^2\right] - 10\left(\frac{u}{5} - u^3 - v^5\right)\exp\left[-u^2 - v^2\right]$ $-\frac{1}{3}\exp\left[-(u+1)^2 - v^2\right]$ where $u = x - x_c$; $v = y - y_c$